My name is Subhashish Chattopadhyay. I have been teaching for IIT-JEE, Various International Exams (such as IMO [International Mathematics Olympiad], IPhO [International Physics Olympiad], IChO [International Chemistry Olympiad]), IGCSE (IB), CBSE, I.Sc, Indian State Board exams such as WB-Board, Karnataka PU-II etc since 1989. As I write this book in 2016, it is my 25th year of teaching. I was a Visiting Professor to BARC Mankhurd, Chembur, Mumbai, Homi Bhabha Centre for Science Education (HBCSE) Physics Olympics camp BARC Campus.
I am Life Member of ...

- IAPT (Indian Association of Physics Teachers)
- IPA (Indian Physics Association)
- AMTI (Association of Mathematics Teachers of India)
- National Human Rights Association
- Men’s Rights Movement (India and International)
- MGTOW Movement (India and International)

And also of

IACT (Indian Association of Chemistry Teachers)

The selection for National Camp (for Official Science Olympiads - Physics, Chemistry, Biology, Astronomy) happens in the following steps ....

1 ) NSEP (National Standard Exam in Physics) and NSEC (National Standard Exam in Chemistry) held around 24th November. Approx 35,000 students appear for these exams every year. The exam fees is Rs 100 each. Since 1998 the IIT JEE toppers have been topping these exams and they get to know their rank/performance ahead of others.

2 ) INPhO (Indian National Physics Olympiad) and INChO (Indian National Chemistry Olympiad). Around 300 students in each subject are allowed to take these exams. Students coming from outside cities are paid fair from the Govt of India.

3 ) The Top 35 students of each subject are invited at HBCSE (Homi Bhabha Center for Science Education) Mankhurd, near Chembur, BARC, Mumbai. After a 2-3 weeks camp the top 5 are selected to represent India. The flight tickets and many other expenses are taken care by Govt of India.

Since last 50 years there has been no dearth of “Good Books“. Those who are interested in studies have been always doing well. This e-Book does not intend to replace any standard text book. These topics are very old and already standardized.
There are 3 kinds of Text Books

- The thin Books - Good students who want more details are not happy with these. Average students who need more examples are not happy with these. Most students who want to “Cram” quickly and pass somehow find the thin books “good” as they have to read less !!

- The Thick Books - Most students do not like these, as they want to read as less as possible. Average students are “busy” with many other things and have no time to read all these.

- The Average sized Books - Good students do not get all details in any one book. Most bad students do not want to read books of “this much thickness“ also !!

We know there can be no shoe that’s fits in all.

Printed books are not e-Books! Can’t be downloaded and kept in hard-disc for reading “later” ……..

So if you read this book later, you will get all kinds of examples in a single place. This becomes a very good “Reference Material”. I sincerely wish that all find this “very useful”.

Students who do not practice lots of problems, do not do well. The rules of “doing well” had never changed …. Will never change !
After 2016 CBSE Mathematics exam, lots of students complained that the paper was tough!

CBSE assures remedial measures for tricky and tough Class XII Math paper

After several students claimed that the Central Board of Secondary Education (CBSE) Class XII board Mathematics examination paper was ‘tricky’ and tough, the board has issued a clarification on remedial measures which are likely to be taken before evaluation.

The CBSE says that feedback received from various stakeholders like students, subject teachers and examiners will be put before the committee of subject experts.
New Delhi: A senior Congress member on Thursday raised the issue of the tough mathematics question paper in the ongoing CBSE board examinations and asked the government to consider the issue "seriously".
In March 2016, students of Karnataka PU-II also complained the same, regarding standard 12 (PU-II Mathematics Exam). Even though the Math Paper was identical to previous year, most students had not even solved the 2015 Question Paper.

These complaints are not new. In fact since last 40 years, (since my childhood), I always see this; every year the same setback, same complain!

In this e-Book I am trying to solve this problem. Those students who practice can learn.

No one can help those who are not studying, or practicing.
A very polite request:

I wish these e-Books are read only by Boys and Men. Girls and Women, better read something else; learn from somewhere else.
We all know that in the species “Homo Sapiens”, males are bigger than females. The reasons are explained in standard 10, or 11 (high school) Biology texts. This shape or size, influences all of our culture. Before we recall / understand the reasons once again, let us see some random examples of the influence.

Random - 1

If there is a Road rage, then who all fight? (generally?). Imagine two cars driven by adult drivers. Each car has a woman of similar age as that of the Man. The cars “touch” or “some issue happens”. Who all comes out and fights? Who all are most probable to drive the cars?

( Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win )

Random - 2

Heavy metal music artists are all Men. Metallica, Black Sabbath, Motley Crue, Megadeth, Motorhead, AC/DC, Deep Purple, Slayer, Guns & Roses, Led Zeppelin, Aerosmith….. the list can be in thousands. All these are grown-up Boys, known as Men.

( Men strive for perfection. Men are eager to excel. Men work hard. Men want to win. )
Random - 3

Apart from Marie Curie, only one more woman got Nobel Prize in Physics. (Maria Goeppert Mayer - 1963). So, ... almost all are men.


Random - 4

The best Tabla Players are all Men.

Random - 5

History is all about, which Kings ruled. Kings, their men, and Soldiers went for wars. History is all about wars, fights, and killings by men.

Boys start fighting from school days. Girls do not fight like this

( Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win. )
Random - 6

The highest award in Mathematics, the “Fields Medal” is around since decades. Till date only one woman could get that. (Maryam Mirzakhani - 2014). So, almost all are men.


Random - 7

Actor is a gender neutral word. Could the movie like “Top Gun” be made with Female actors? The best pilots, astronauts, Fighters are all Men.
In my childhood had seen a movie named “The Tower in Inferno“. In the movie when the tall tower is in fire, women were being saved first, as only one lift was working....

Many decades later another movie is made. A box office hit. “The Titanic“. In this also .... As the ship is sinking women are being saved. Men are disposable. Men may get their turn later...

Movies are not training programs. Movies do not teach people what to do, or not to do. Movies only reflect the prevalent culture. Men are disposable, is the culture in the society. Knowingly, unknowingly, the culture is depicted in Movies, Theaters, Stories, Poems, Rituals, etc. I or you can’t write a story, or make a movie in which after a minor car accident the Male passengers keep seating in the back seat, while the both the women drivers come out of the car and start fighting very bitterly on the road. There has been no story in this world, or no movie made, where after an accident or calamity, Men are being helped for safety first, and women are told to wait.
Random - 9

Artists generally follow the prevalent culture of the Society. In paintings, sculptures, stories, poems, movies, cartoon, Caricatures, knowingly / unknowingly, “the prevalent Reality “ is depicted. The opposite will not go well with people. If deliberately “the opposite “ is shown then it may only become a special art, considered as a special mockery.

Random - 10

Men go to “girl / woman’s house” to marry / win, and bring her to his home. That is a sort of winning her. When a boy gets a “ Girl-Friend “, generally he and his friends consider that as an achievement. The boy who “got / won “ a girl-friend feels proud. His male friends feel, jealous, competitive and envious. Millions of stories have been written on these themes. Lakhs of movies show this. Boys / Men go for “bike race “, or say “ Car Race “, where the winner “gets“ the most beautiful girl of the college.

( Men want to excel. Men are eager to fight, eager to rule, eager for war. Men want to drive. Men want to win. )

Prithviraj Chauhan ‘ went ` to “pickup “ or “ abduct “ or “ win “ or “ bring “ his love. There was a Hindi movie (hit) song ... “ Pasand ho jaye, to ghar se utha laye “. It is not other way round. Girls do not go to Boy’s house or man’s house to marry. Nor the girls go in a gang to “pick-up “ the boy / man and bring him to their home / place / den.
Rich people; often are very hard working. Successful business men, establish their business (empire), amass lot of wealth, with lot of difficulty. Lots of sacrifice, lots of hard work, gets into this. Rich people’s wives had no contribution in this wealth creation. Women are smart, and successful upto the extent to choose the right/rich man to marry. So generally what happens in case of Divorces? Search the net on “most costly divorces” and you will know. The women; (who had no contribution at all, in setting up the business / empire), often gets in Billions, or several Millions in divorce settlements.

Ted Danson & Casey Coates -- $30 million

Ted Danson’s claim to fame is undoubtedly his decade-long stint as Sam Malone on NBC’s celebrated sitcom Cheers. While he did other TV shows and movies, he will always be known as the bartender of that place where everybody knows your name. He met his future first bride, Casey, a designer, in 1976 while doing Erhard Seminars Training.

Ten years his senior, she suffered a paralyzing stroke while giving birth to their first child in 1979. In order to nurse her back to health, Danson took a break from acting for six months. But after two children and 16 years of marriage, the infatuation fell to pieces. Danson had started seeing Whoopi Goldberg while filming the comedy, Made in America and this precipitated the 1992 divorce. Casey got $30 million for her trouble.


It was Boys and Men, who brought the girls / women home. The Laws are biased, completely favoring women. The men are paying for their own mistakes.

See [https://zookeepersblog.wordpress.com/biased-laws/](https://zookeepersblog.wordpress.com/biased-laws/)

( Man brings the Woman home. When she leaves, takes away her share of big fortune! )

A standardized test of Intelligence will never be possible. It never happened before, nor ever will happen in future; where the IQ test results will be acceptable by all. In the net there are thousands of charts which show that the intelligence scores of girls / women are lesser. Debates of Trillion words, does not improve performance of Girls.
I am not wasting a single second debating or discussing with anyone, on this. I am simply accepting ALL the results. IQ is only one of the variables which is required for success in life. Thousands of books have been written on “Networking Skills”, EQ (Emotional Quotient), Drive, Dedication, Focus, “Tenacity towards the end goal ‘...’ etc. In each criteria, and in all together, women (in general) do far worse than men. Bangalore is known as “..... capital of India”. [Fill in the blanks]. The blanks are generally filled as “Software Capital”, “IT Capital”, “Startup Capital”, etc. I am member in several startup eco-systems / groups. I have attended hundreds of meetings, regarding “technology startups “, or “idea startups “. These meetings have very few women. Starting up new companies are all “Men’s Game” / “Men’s business “. Only in Divorce settlements women will take their goodies, due to Biased laws. There is no dedication, towards wealth creation, by women.

Random - 13

Many men, as fathers, very unfortunately treat their daughters as “Princess”. Every “non-performing “ woman / wife was “ princess daughter “ of some loving father. Pampering the girls, in name of “equal opportunity “, or “women empowerment “, have led to nothing.
There can be thousands of more such random examples, where “Bigger Shape / size “ of males have influenced our culture, our Society. Let us recall the reasons, that we already learned in standard 10 - 11, Biology text Books. In humans, women have a long gestation period, and also spends many years (almost a decade) to grow, nourish, and stabilize the child. (Million years of habit) Due to survival instinct Males want to inseminate. Boys and Men fight for the “facility (of womb + care) “ the girl / woman may provide. Bigger size for males, has a winning advantage. Whoever wins, gets the “woman / facility “. The male who is of “Bigger Size “, has an advantage to win.... Leading to Natural selection over millions of years. In general “Bigger Males “; the “fighting instinct “in men; have led to wars, and solving tough problems (Mathematics, Physics, Technology, startups of new businesses, Wealth creation, Unreasonable attempts to make things [such as planes], Hard work ....)

So let us see the IIT-JEE results of girls. Statistics of several years show that there are around 17, (or less than 20) girls in top 1000 ranks, at all India level. Some people will yet not understand the performance, till it is said that … year after year we have around 980 boys in top 1000 ranks. Generally we see only 4 to 5 girls in top 500. In last 50 years not once any girl topped in IIT-JEE advanced. Forget about Single digit ranks, double digit ranks by girls have been extremely rare. It is all about “good boys“, “hard working“, “focused“, “Bel-esprit “ boys.

In 2015, Only 2.6% of total candidates who qualified are girls (upto around 12,000 rank). while 20% of the Boys, amongst all candidates qualified. The Total number of students who appeared for the exam were around 1.4 million for IIT-JEE main. Subsequently 1.2 lakh (around 120 thousands) appeared for IIT-JEE advanced.

IIT-JEE results and analysis, of many years is given at https://zookeepersblog.wordpress.com/iit-jee-iseet-main-and-advanced-results/

In Bangalore it is rare to see a girl with rank better than 1000 in IIT-JEE advanced. We hardly see 6-7 boys with rank better than 1000. Hardly 2-3 boys get a rank better than 500.

See http://skmclasses.weebly.com/everybody-knows-so-you-should-also-know.html

Thousands of people are exposing the heinous crimes that Motherly Women are doing, or Female Teachers are committing. See https://www.facebook.com/WomenCriminals/
In several countries or rather in several regions of the world, family system has collapsed, due to bad nature and naughty acts of women. Particularly in Britain and America, almost 50% people are alone, lonely, separated, divorced or failed marriages. In 2013, 48% children were born out of wedlock. It was projected that by 2016, more than 51% children will be born, to unmarried mothers. In these developed countries “paternity fraud” by women, are close to 20%. You can see several articles in the net, and in wikipedia etc. This means 1 out of 5 children are calling a wrong man as dad. The lonely, alone “mothers” are frustrated. They see the children as burden. Love in the Society in general is lost, long time ago. The types of “Mothers “ and “Women “ we have now.........
By now if you have assumed that Indian women are not doing any crime then please become friends with MRA Guri [https://www.facebook.com/profile.php?id=100004138754180](https://www.facebook.com/profile.php?id=100004138754180)

He has dedicated his life to expose Indian Criminals
Spoon Feeding Series - Complex or Imaginary Numbers

Question

Express the given complex number in the form $a + ib$: $(5i)(-\frac{3}{5}i)$

Answer

\[ (5i)(-\frac{3}{5}i) = -5 \times \frac{3}{5} \times i \times i \]

\[ = -3i^2 \]

\[ = -3(-1) \quad [i^2 = -1] \]

\[ = 3 \]

Question

Express the given complex number in the form $a + ib$: $i^9 + i^{19}$

Answer

\[ i^9 + i^{19} = i^{4 \times 2 + 1} + i^{4 \times 4 + 3} \]

\[ = (i^4)^2 \cdot i + (i^4)^4 \cdot i \]

\[ = 1 \times i + 1 \times (-i) \quad [i^4 = 1, \quad i^3 = -i] \]

\[ = i + (-i) \]

\[ = 0 \]
Express the given complex number in the form $a + ib$: $i^{-39}$

**Answer**

\[
i^{-39} = i^{-5 \cdot 7 + 4} = \left(i^4\right)^{-9} \cdot i^3 = (1)^{-9} \cdot i^3 = \frac{1}{i^3} = \frac{1}{-i} = \frac{-i}{-1} = i
\]

$[i^4 = 1, i^3 = -i, i^2 = -1]$

---

Express the given complex number in the form $a + ib$: $3(7 + i7) + i(7 + i7)$

**Answer**

\[
3(7 + i7) + i(7 + i7) = 21 + 21i + 7i + 7i^2 = 21 + 28i + 7 \times (-1) = 14 + 28i
\]

$[\therefore i^2 = -1]$

---

Express the given complex number in the form $a + ib$: $(1 - i) - (-1 + i6)$

**Answer**

\[
(1 - i) - (-1 + i6) = 1 - i + 1 - 6i = 2 - 7i
\]
Question

Express the given complex number in the form $a + ib$: \( \left( \frac{1}{5} + i \frac{2}{5} \right) - \left( 4 + i \frac{5}{2} \right) \)

Answer

\[
\begin{align*}
\left( \frac{1}{5} + i \frac{2}{5} \right) - \left( 4 + i \frac{5}{2} \right) \\
= \frac{1}{5} + i \frac{2}{5} - 4 - i \frac{5}{2} \\
= \left( \frac{1}{5} - 4 \right) + i \left( \frac{2}{5} - \frac{5}{2} \right) \\
= -\frac{19}{5} + i \left( -\frac{21}{10} \right) \\
= -\frac{19}{5} - \frac{21}{10} i
\end{align*}
\]

Question

Express the given complex number in the form $a + ib$: \[ \left[ \left( \frac{1}{3} + i \frac{7}{3} \right) + \left( 4 + i \frac{1}{3} \right) \right] - \left( \frac{4}{3} + i \right) \]

Answer

\[
\begin{align*}
\left[ \left( \frac{1}{3} + i \frac{7}{3} \right) + \left( 4 + i \frac{1}{3} \right) \right] - \left( \frac{4}{3} + i \right) \\
= \frac{1}{3} + i \frac{7}{3} + 4 + i \frac{1}{3} - \frac{4}{3} - i \\
= \left( \frac{1}{3} + 4 + \frac{4}{3} \right) + i \left( \frac{7}{3} + \frac{1}{3} - 1 \right) \\
= \frac{17}{3} + i \frac{5}{3}
\end{align*}
\]
Question

Express the given complex number in the form $a + ib$: $(1 - i)^4$

Answer

$$(1 - i)^4 = \left[(1 - i)^2\right]^2$$
$$= \left[1^2 + i^2 - 2i\right]^2$$
$$= \left[1 - 1 - 2i\right]^2$$
$$= (-2i)^2$$
$$= (-2i) \times (-2i)$$
$$= 4i^2 = -4 \quad \text{[} i^2 = -1 \text{]}$$

Question

Express the given complex number in the form $a + ib$: $\left(\frac{1}{3} + 3i\right)^3$

Answer

$$\left(\frac{1}{3} + 3i\right)^3 = \left(\frac{1}{3}\right)^3 + (3i)^3 + 3\left(\frac{1}{3}\right)(3i)\left(\frac{1}{3} + 3i\right)$$
$$= \frac{1}{27} + 27i^3 + 3\left(\frac{1}{3} + 3i\right)$$
$$= \frac{1}{27} + 27(-i) + i + 9i^2 \quad \text{[} i^3 = -i \text{]}$$
$$= \frac{1}{27} - 27i + i - 9 \quad \text{[} i^2 = -1 \text{]}$$
$$= \left(\frac{1}{27} - 9\right) + i(-27 + 1)$$
$$= \frac{-242}{27} - 26i$$
Question

Express the given complex number in the form $a + ib$:

$$\left( -2 \frac{1}{3} i \right)^3$$

Answer

$$\left( -2 \frac{1}{3} i \right)^3 = (-1)^3 \left( 2 + \frac{1}{3} i \right)^3$$

$$= - \left[ \left( 2^3 + \left( \frac{i}{3} \right)^3 + 3 \left( 2 \right) \left( \frac{i}{3} \right) \left( 2 + \frac{i}{3} \right) \right) \right]$$

$$= - \left[ 8 + \frac{i^3}{27} + 2i \left( 2 + \frac{i}{3} \right) \right]$$

$$= - \left[ 8 - \frac{i}{27} + 4i \left( \frac{2i}{3} \right) \right] \quad \text{[} i^3 = -i \text{]}$$

$$= - \left[ 8 - \frac{i}{27} + 4i \left( \frac{2}{3} \right) \right] \quad \text{[} i^2 = -1 \text{]}$$

$$= - \left[ \frac{22}{3} + \frac{107i}{27} \right]$$

$$= - \frac{22}{3} - \frac{107i}{27}$$
Question

Find the multiplicative inverse of the complex number $4 - 3i$

Answer

Let $z = 4 - 3i$

Then, $\bar{z} = 4 + 3i$ and $|z|^2 = 4^2 + (-3)^2 = 16 + 9 = 25$

Therefore, the multiplicative inverse of $4 - 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{4 + 3i}{25} = \frac{4}{25} + \frac{3}{25}i$$

Question

Find the multiplicative inverse of the complex number $\sqrt{5} + 3i$

Answer

Let $z = \sqrt{5} + 3i$

Then, $\bar{z} = \sqrt{5} - 3i$ and $|z|^2 = (\sqrt{5})^2 + 3^2 = 5 + 9 = 14$

Therefore, the multiplicative inverse of $\sqrt{5} + 3i$ is given by

$$z^{-1} = \frac{\bar{z}}{|z|^2} = \frac{\sqrt{5} - 3i}{14} = \frac{\sqrt{5}}{14} - \frac{3i}{14}$$
Question

Find the multiplicative inverse of the complex number \(-i\)

Answer

Let \(z = -i\)

Then, \(\overline{z} = i\) and \(|z|^2 = 1^2 = 1\)

Therefore, the multiplicative inverse of \(-i\) is given by

\[
 z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{i}{1} = i
\]

Question

Express the following expression in the form of \(a + ib\).

\[
 \frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}
\]

Answer

\[
 \frac{(3+i\sqrt{5})(3-i\sqrt{5})}{(\sqrt{3}+\sqrt{2}i)-(\sqrt{3}-i\sqrt{2})}
\]

\[
 = \frac{(3)^2-(i\sqrt{5})^2}{\sqrt{3}+\sqrt{2}i-\sqrt{3}+\sqrt{2}i}
\]

\[
 = \frac{9-5i^2}{2\sqrt{2}i}
\]

\[
 = \frac{9-5(-1)}{2\sqrt{2}i}
\]

\[
 = \frac{9+5i}{2\sqrt{2}i} \times \frac{i}{i}
\]

\[
 = \frac{9+5}{2\sqrt{2}i} \times i
\]

\[
 = \frac{14i}{2\sqrt{2}i} = \frac{7}{\sqrt{2}}
\]
Question

Find the modulus and the argument of the complex number \( z = -1 - i\sqrt{3} \)

Answer

Let \( r \cos \theta = -1 \) and \( r \sin \theta = -\sqrt{3} \)

On squaring and adding, we obtain

\[
(r \cos \theta)^2 + (r \sin \theta)^2 = (-1)^2 + (-\sqrt{3})^2
\]

\[
\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 3
\]

\[
\Rightarrow r^2 = 4
\]

\[
\Rightarrow r = \sqrt{4} = 2
\]

[Conventionally, \( r > 0 \)]

\[\therefore\text{Modulus} = 2\]

\[\therefore 2 \cos \theta = -1 \text{ and } 2 \sin \theta = -\sqrt{3}\]

\[
\Rightarrow \cos \theta = -\frac{1}{2} \text{ and } \sin \theta = -\frac{\sqrt{3}}{2}
\]
Since both the values of \( \sin \theta \) and \( \cos \theta \) are negative and \( \sin \theta \) and \( \cos \theta \) are negative in the III quadrant,

\[
\text{Argument} = -\left( \pi - \frac{\pi}{3} \right) = -\frac{2\pi}{3}
\]

Thus, the modulus and argument of the complex number \(-1 - \sqrt{3}i\) are 2 and \(-\frac{2\pi}{3}\) respectively.

**Question**

Find the modulus and the argument of the complex number \( z = -\sqrt{3} + i \)

**Answer**

\( z = -\sqrt{3} + i \)

Let \( r \cos \theta = -\sqrt{3} \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain

\[
r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left( -\sqrt{3} \right)^2 + 1^2
\]

\[
\Rightarrow r^2 = 3 + 1 = 4
\]

\[
\Rightarrow r = \sqrt{4} = 2
\]

Conventionally, \( r > 0 \)

\[\therefore \text{ Modulus } = 2\]

\[
\therefore 2 \cos \theta = -\sqrt{3} \text{ and } 2 \sin \theta = 1
\]

\[
\Rightarrow \cos \theta = \frac{-\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2}
\]

\[
\therefore \theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}
\]

[As \( \theta \) lies in the II quadrant]

Thus, the modulus and argument of the complex number \(-\sqrt{3} + i\) are 2 and \(\frac{5\pi}{6}\) respectively.
Question

Convert the given complex number in polar form: \(1 - i\)

Answer

\(1 - i\)

Let \(r \cos \theta = 1\) and \(r \sin \theta = -1\)

On squaring and adding, we obtain

\[
r^2 \cos^2 \theta + r^2 \sin^2 \theta = 1^2 + (-1)^2
\]

\[
\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1
\]

\[
\Rightarrow r^2 = 2
\]

\[
\Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0] \]

\[
\therefore \sqrt{2} \cos \theta = 1 \text{ and } \sqrt{2} \sin \theta = -1
\]

\[
\Rightarrow \cos \theta = \frac{1}{\sqrt{2}} \text{ and } \sin \theta = -\frac{1}{\sqrt{2}}
\]

\[
\therefore \theta = -\frac{\pi}{4} \quad \text{[As } \theta \text{ lies in the IV quadrant]}
\]

\[
\therefore 1 - i = r \cos \theta + ir \sin \theta = \sqrt{2} \cos \left(-\frac{\pi}{4}\right) + i \sqrt{2} \sin \left(-\frac{\pi}{4}\right) = \sqrt{2} \left[\cos \left(-\frac{\pi}{4}\right) + i \sin \left(-\frac{\pi}{4}\right)\right]
\]

This is the required polar form.
Question

Convert the given complex number in polar form: \(-1 + i\)

Answer

\(-1 + i\)

Let \(r \cos \theta = -1\) and \(r \sin \theta = 1\)

On squaring and adding, we obtain

\[r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + 1^2\]

\[\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1\]

\[\Rightarrow r^2 = 2\]

\[\Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0\text{]}\]

\[\therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1\]

\[\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}\]

\[\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \text{[As } \theta \text{ lies in the II quadrant]}\]

It can be written,

\[\therefore -1 + i = r \cos \theta + ir \sin \theta = \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right)\]

This is the required polar form.
Question

Convert the given complex number in polar form: $-1 - i$

Answer

$-1 - i$

Let $r \cos \theta = -1$ and $r \sin \theta = -1$

On squaring and adding, we obtain

\[
r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-1)^2 + (-1)^2
\]

\[
\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1
\]

\[
\Rightarrow r^2 = 2
\]

\[
\Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0]\]

\[
\therefore \sqrt{2} \cos \theta = -1 \quad \text{and} \quad \sqrt{2} \sin \theta = -1
\]

\[
\Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \quad \text{and} \quad \sin \theta = -\frac{1}{\sqrt{2}}
\]

\[
\therefore \theta = -\left(\pi - \frac{\pi}{4}\right) = -\frac{3\pi}{4} \quad \text{[As } \theta \text{ lies in the III quadrant]}\]

\[
\therefore -1 - i = r \cos \theta + ir \sin \theta = \sqrt{2} \cos \left(-\frac{3\pi}{4}\right) + i\sqrt{2} \sin \left(-\frac{3\pi}{4}\right) = \sqrt{2} \left(\cos \left(-\frac{3\pi}{4}\right) + i \sin \left(-\frac{3\pi}{4}\right)\right)
\]

This is the required polar form.

Question

Convert the given complex number in polar form: $-3$

Answer

$-3$

Let $r \cos \theta = -3$ and $r \sin \theta = 0$

On squaring and adding, we obtain
\[ r^2 \cos^2 \theta + r^2 \sin^2 \theta = (-3)^2 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 9 \]
\[ \Rightarrow r^2 = 9 \]
\[ \Rightarrow r = \sqrt{9} = 3 \quad \text{[Conventionally, } r > 0 \text{]} \]
\[ \therefore 3 \cos \theta = -3 \text{ and } 3 \sin \theta = 0 \]
\[ \Rightarrow \cos \theta = -1 \text{ and } \sin \theta = 0 \]
\[ \therefore \theta = \pi \]
\[ \therefore -3 = r \cos \theta + ir \sin \theta = 3 \cos \pi + i \sin \pi = 3 \left( \cos \pi + i \sin \pi \right) \]
This is the required polar form.

**Question**

Convert the given complex number in polar form: \( \sqrt{3} + i \)

**Answer**

\( \sqrt{3} + i \)

Let \( r \cos \theta = \sqrt{3} \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain

\[ r^2 \cos^2 \theta + r^2 \sin^2 \theta = \left( \sqrt{3} \right)^2 + 1^2 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 3 + 1 \]
\[ \Rightarrow r^2 = 4 \]
\[ \Rightarrow r = \sqrt{4} = 2 \quad \text{[Conventionally, } r > 0 \text{]} \]
\[ \therefore 2 \cos \theta = \sqrt{3} \text{ and } 2 \sin \theta = 1 \]
\[ \Rightarrow \cos \theta = \frac{\sqrt{3}}{2} \text{ and } \sin \theta = \frac{1}{2} \]
\[ \therefore \theta = \frac{\pi}{6} \quad \text{[As } \theta \text{ lies in the 1 quadrant]} \]
This is the required polar form

Question

Convert the given complex number in polar form: \( i \)

Answer

\( i \)

Let \( r \cos \theta = 0 \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain

\[
r^2 \cos^2 \theta + r^2 \sin^2 \theta = 0^2 + 1^2
\]

\[
\Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 1
\]

\[
\Rightarrow r^2 = 1
\]

\[
\Rightarrow r = \sqrt{1} = 1 \quad [\text{Conventionally, } r > 0]
\]

\[
\therefore \cos \theta = 0 \text{ and } \sin \theta = 1
\]

\[
\therefore \theta = \frac{\pi}{2}
\]

\[
\therefore i = r \cos \theta + ir \sin \theta = \cos \frac{\pi}{2} + i \sin \frac{\pi}{2}
\]

This is the required polar form.
Question

Evaluate:

\[
\left[ i^{18} + \left( \frac{1}{i} \right)^{25} \right]^3
\]

Answer

\[
\left[ i^{18} + \left( \frac{1}{i} \right)^{25} \right]^3
= \left[ i^{18} + i^{-25} \right]^3
= \left[ (i^4)^4 \cdot i^2 + \frac{1}{(i^4)^6 \cdot i} \right]^3
= \left[ i^2 + \frac{1}{i} \right]^3
= \left[ -1 + \frac{1}{i} \cdot -i \right]^3
= \left[ -1 + \frac{i^2}{i} \right]^3
= \left[ -1 + i \right]^3
= (-1)^3 [1 + i]^3
\]

\[
= -\left[ 1^3 + i^3 + 3 \cdot i \cdot (1 + i) \right]
= -\left[ 1 + i^3 + 3i + 3i^2 \right]
= -\left[ 1 - i + 3i - 3 \right]
= -\left[ -2 + 2i \right]
= 2 - 2i
\]
Question

For any two complex numbers $z_1$ and $z_2$, prove that

$$\text{Re} \left( z_1 z_2 \right) = \text{Re} \, z_1 \, \text{Re} \, z_2 - \text{Im} \, z_1 \, \text{Im} \, z_2$$

Answer

Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$\therefore z_1 z_2 = (x_1 + iy_1)(x_2 + iy_2)$

$$= x_1(x_2 + iy_2) + iy_1(x_2 + iy_2)$$

$$= x_1 x_2 + i(x_1 y_2 + y_1 x_2) + i^2 y_1 y_2$$

$$= x_1 x_2 + i(x_1 y_2 + y_1 x_2) - y_1 y_2$$

$$= \left( x_1 x_2 - y_1 y_2 \right) + i \left( x_1 y_2 + y_1 x_2 \right)$$

$\Rightarrow \text{Re} \left( z_1 z_2 \right) = x_1 x_2 - y_1 y_2$

$\Rightarrow \text{Re} \left( z_1 z_2 \right) = \text{Re} \, z_1 \, \text{Re} \, z_2 - \text{Im} \, z_1 \, \text{Im} \, z_2$

Hence, proved.
Question

Reduce \( \left( \frac{1}{1-4i} - \frac{2}{1+i} \right) \left( \frac{3-4i}{5+i} \right) \) to the standard form.

Answer

\[
\left( \frac{1}{1-4i} - \frac{2}{1+i} \right) \left( \frac{3-4i}{5+i} \right) = \left( \frac{1+i}{1-4i} \right) \left( \frac{1-4i}{1+i} \right) \left( \frac{3-4i}{5+i} \right)
\]

\[
= \frac{1+i-2+8i}{1+i-4i+4i^2} \cdot \frac{3-4i}{5+i} = \frac{-1+9i}{5-3i} \cdot \frac{3-4i}{5+i}
\]

\[
= \frac{-3+4i+27i-36i^2}{25+5i-15i-3i^2} = \frac{33+31i}{28-10i} = \frac{33+31i}{2(14-5i)}
\]

\[
= \frac{33+31i}{2(14-5i)} \times \frac{(14+5i)}{(14+5i)} = \frac{462+165i+434i+155i^2}{2[(14)^2-(5i)^2]} = \frac{307+599i}{2(196-25i)}
\]

\[
= \frac{307+599i}{2(221)} = \frac{307+599i}{442} = \frac{307}{442} + \frac{599i}{442}
\]

This is the required standard form.
Question

If \( x - iy = \frac{a - ib}{c - id} \) prove that \( (x^2 + y^2)^2 = \frac{a^2 + b^2}{c^2 + d^2} \).

Answer

\[
\begin{align*}
  x - iy &= \frac{a - ib}{c - id} \\
  &= \frac{a - ib}{c - id} \times \frac{c + id}{c + id} \quad \text{[On multiplying numerator and denominator by \((c + id)]\}} \\
  &= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2} \\
  \therefore (x - iy)^2 &= \frac{(ac + bd) + i(ad - bc)}{c^2 + d^2} \\
  \Rightarrow x^2 - y^2 - 2ixy &= \frac{ac + bd + i(ad - bc)}{c^2 + d^2} \\
  \text{On comparing real and imaginary parts, we obtain} \\
  x^2 - y^2 &= \frac{ac + bd}{c^2 + d^2}, \quad 2xy = \frac{ad - bc}{c^2 + d^2} \quad (1)
\end{align*}
\]
\[
(x^2 + y^2)^2 = (x^2 - y^2)^2 + 4x^2y^2
\]

\[
= \left(\frac{ac + bd}{c^2 + d^2}\right)^2 + \left(\frac{ad - bc}{c^2 + d^2}\right)^2 \quad \text{[Using (1)]}
\]

\[
= \frac{a^2c^2 + b^2d^2 + 2acbd + a^2d^2 + b^2c^2 - 2adbc}{(c^2 + d^2)^2}
\]

\[
= \frac{a^2c^2 + b^2d^2 + a^2d^2 + b^2c^2}{(c^2 + d^2)^2}
\]

\[
= \frac{a^2(c^2 + d^2) + b^2(c^2 + d^2)}{(c^2 + d^2)^2}
\]

\[
= \frac{(c^2 + d^2)(a^2 + b^2)}{(c^2 + d^2)^2}
\]

\[
= \frac{a^2 + b^2}{c^2 + d^2}
\]

Hence, proved.
Question

Convert the following in the polar form:

(i) \( \frac{1+7i}{(2-i)^2} \), (ii) \( \frac{1+3i}{1-2i} \)

Answer

(i) Here,

\[
\begin{align*}
\frac{1+7i}{(2-i)^2} &= \frac{1+7i}{4-4i+i^2} = \frac{1+7i}{4-1-4i} \\
&= \frac{1+7i}{3-4i} \times \frac{3+4i}{3+4i} = \frac{3+4i+21i+28i^2}{3^2+4^2} \\
&= \frac{3+4i+21i-28}{3^2+4^2} = \frac{-25+25i}{25} \\
&= -1+i
\end{align*}
\]

Let \( r \cos \theta = -1 \) and \( r \sin \theta = 1 \)
On squaring and adding, we obtain
\[ r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2 \]
\[ \Rightarrow r^2 = 2 \]
\[ \Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0 \text{]} \]
\[ \therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1 \]
\[ \Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}} \]
\[ \therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \text{[As } \theta \text{ lies in II quadrant]} \]
\[ \therefore z = r \cos \theta + i \, r \sin \theta \]
\[ = \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \]

This is the required polar form.

Question

(ii) Here,
\[ z = \frac{1+3i}{1-2i} \]
\[ = \frac{1+3i}{1-2i} \times \frac{1+2i}{1+2i} \]
\[ = \frac{1+2i+3i-6}{1+4} \]
\[ = \frac{-5+5i}{5} = -1+i \]

Let \( r \cos \theta = -1 \) and \( r \sin \theta = 1 \)

On squaring and adding, we obtain
\[ r^2 (\cos^2 \theta + \sin^2 \theta) = 1 + 1 \]
\[ \Rightarrow r^2 (\cos^2 \theta + \sin^2 \theta) = 2 \]
\[ \Rightarrow r^2 = 2 \]
\[ \Rightarrow r = \sqrt{2} \quad \text{[Conventionally, } r > 0 \text{]} \]
\[ r = \sqrt{2} \quad \text{[Conventionally, } r > 0] \]
\[ \therefore \sqrt{2} \cos \theta = -1 \text{ and } \sqrt{2} \sin \theta = 1 \]
\[ \Rightarrow \cos \theta = -\frac{1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}} \]
\[ \therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \text{[As } \theta \text{ lies in II quadrant]} \]
\[ z = r \cos \theta + i r \sin \theta \]
\[ = \sqrt{2} \cos \frac{3\pi}{4} + i \sqrt{2} \sin \frac{3\pi}{4} = \sqrt{2} \left( \cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4} \right) \]

This is the required polar form.
Question

If \( z_1 = 2 - i, \ z_2 = 1 + i \), find \( \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| \)

Answer

\[ z_1 = 2 - i, \ z_2 = 1 + i \]

\[ \therefore \left| \frac{z_1 + z_2 + 1}{z_1 - z_2 + i} \right| = \left| \frac{(2-i)+(1+i)+1}{(2-i)-(1+i)+1} \right| \]

\[ = \left| \frac{4}{2-2i} \right| = \left| \frac{4}{2(1-i)} \right| \]

\[ = \frac{2}{1-i} \times \frac{1+i}{1+i} = \left| \frac{2(1+i)}{1^2 - i^2} \right| \]

\[ = \frac{2(1+i)}{1+1} \quad \left[ i^2 = -1 \right] \]

\[ = \frac{2(1+i)}{2} \]

\[ = |1+i| = \sqrt{1^2 + 1^2} = \sqrt{2} \]
Question

\[
\frac{(x+i)^2}{2x^2+1}, \text{ prove that } a^2 + b^2 = \frac{(x^2+1)^2}{(2x+1)^2}
\]

Answer

\[
a + ib = \frac{(x + i)^2}{2x^2 + 1} = \frac{x^2 + i^2 + 2xi}{2x^2 + 1} = \frac{x^2 - 1 + i2x}{2x^2 + 1} = \frac{x^2 - 1}{2x^2 + 1} + i \left( \frac{2x}{2x^2 + 1} \right)
\]

On comparing real and imaginary parts, we obtain

\[
a = \frac{x^2 - 1}{2x^2 + 1} \quad \text{and} \quad b = \frac{2x}{2x^2 + 1}
\]
\[ \therefore a^2 + b^2 = \left( \frac{x^2 - 1}{2x^2 + 1} \right)^2 + \left( \frac{2x}{2x^2 + 1} \right)^2 \]
\[ = \frac{x^4 + 1 - 2x^2 + 4x^2}{(2x + 1)^2} \]
\[ = \frac{x^4 + 1 + 2x^2}{(2x^2 + 1)^2} \]
\[ = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2} \]
\[ \therefore a^2 + b^2 = \frac{(x^2 + 1)^2}{(2x^2 + 1)^2} \]

Hence, proved.
Question

Let \( z_1 = 2 - i, \ z_2 = -2 + i \). Find

\[
\begin{align*}
\text{Re}\left( \frac{z_1 z_2}{\overline{z}_1} \right) & \quad \text{and} \quad \text{Im}\left( \frac{1}{z_1 \overline{z}_1} \right) \\
\end{align*}
\]

(i)

Answer

\( z_1 = 2 - i, \ z_2 = -2 + i \)

\[
\begin{align*}
\frac{z_1 z_2}{\overline{z}_1} &= (2 - i)(-2 + i) = -4 + 2i + 2i - i^2 = -4 + 4i - (-1) = -3 + 4i \\
\overline{z}_1 &= 2 + i \\
\therefore \quad \frac{z_1 z_2}{\overline{z}_1} &= \frac{-3 + 4i}{2 + i} \\
\end{align*}
\]

On multiplying numerator and denominator by \((2 - i)\), we obtain

\[
\begin{align*}
\frac{z_1 z_2}{\overline{z}_1} &= \frac{(-3 + 4i)(2 - i)}{(2 + i)(2 - i)} = \frac{-6 + 3i + 8i - 4i^2}{2^2 + 1^2} = \frac{-6 + 11i - 4(-1)}{4 + 1} = \frac{-2 + 11i}{5} = \frac{-2}{5} + \frac{11}{5}i \\
\end{align*}
\]

On comparing real parts, we obtain

\[
\text{Re}\left( \frac{z_1 z_2}{\overline{z}_1} \right) = \frac{-2}{5}
\]

(ii)

\[
\begin{align*}
\frac{1}{z_1 \overline{z}_1} &= \frac{1}{(2 - i)(2 + i)} = \frac{1}{(2)^2 + (1)^2} = \frac{1}{5} \\
\end{align*}
\]

On comparing imaginary parts, we obtain

\[
\text{Im}\left( \frac{1}{z_1 \overline{z}_1} \right) = 0
\]
Find the modulus and argument of the complex number \(1 + 2i\) \(1 - 3i\).

Answer

Let \(z = \frac{1 + 2i}{1 - 3i}\), then

\[
z = \frac{1 + 2i}{1 - 3i} \times \frac{1 + 3i}{1 + 3i} = \frac{1 + 3i + 2i + 6i^2}{1 + 3^2} = \frac{1 + 5i + 6(-1)}{1 + 9} = \frac{-5 + 5i}{10} = \frac{-5}{10} + \frac{5i}{10} = \frac{-1}{2} + \frac{1}{2}i.
\]

Let \(z = r \cos \theta + ir \sin \theta\)

i.e., \(r \cos \theta = \frac{-1}{2}\) and \(r \sin \theta = \frac{1}{2}\)

On squaring and adding, we obtain

\[
r^2 \left( \cos^2 \theta + \sin^2 \theta \right) = \left( \frac{-1}{2} \right)^2 + \left( \frac{1}{2} \right)^2
\]

\[\Rightarrow r^2 = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}
\]

\[\Rightarrow r = \frac{1}{\sqrt{2}} \quad \text{[Conventionally, } r > 0]\]

\[\therefore \frac{1}{\sqrt{2}} \cos \theta = \frac{-1}{2} \text{ and } \frac{1}{\sqrt{2}} \sin \theta = \frac{1}{2}
\]

\[\Rightarrow \cos \theta = \frac{-1}{\sqrt{2}} \text{ and } \sin \theta = \frac{1}{\sqrt{2}}
\]

\[\therefore \theta = \pi - \frac{\pi}{4} = \frac{3\pi}{4} \quad \text{[As } \theta \text{ lies in the II quadrant]}\]

Therefore, the modulus and argument of the given complex number are \(\frac{1}{\sqrt{2}}\) and \(\frac{3\pi}{4}\) respectively.
Question

Find the real numbers $x$ and $y$ if $(x - iy)(3 + 5i)$ is the conjugate of $-6 - 24i$.

Answer

Let $z = (x - iy)(3 + 5i)$

$$z = 3x + 5xi - 3yi - 5yi^2 = 3x + 5xi - 3yi + 5y = (3x + 5y) + i(5x - 3y)$$

$\therefore \overline{z} = (3x + 5y) - i(5x - 3y)$

It is given that, $\overline{z} = -6 - 24i$

$\therefore (3x + 5y) - i(5x - 3y) = -6 - 24i$

Equating real and imaginary parts, we obtain

$3x + 5y = -6 \quad \ldots (i)$

$5x - 3y = 24 \quad \ldots (ii)$

Multiplying equation (i) by 3 and equation (ii) by 5 and then adding them, we obtain

$9x + 15y = -18$

$25x - 15y = 120$

$\frac{34x}{34} = 102$

$\therefore x = \frac{102}{34} = 3$

Putting the value of $x$ in equation (i), we obtain

$3(3) + 5y = -6$

$\Rightarrow 5y = -6 - 9 = -15$

$\Rightarrow y = -3$

Thus, the values of $x$ and $y$ are 3 and -3 respectively.
Question

Find the modulus of \( \frac{1+i}{1-i} - \frac{1-i}{1+i} \).

Answer

\[
\frac{1+i}{1-i} - \frac{1-i}{1+i} = \frac{(1+i)^2 - (1-i)^2}{(1-i)(1+i)} = \frac{1+i^2 + 2i - 1 - i^2}{1^2 + 1^2} = \frac{4i}{2} = 2i
\]

\[
\therefore \left| \frac{1+i}{1-i} - \frac{1-i}{1+i} \right| = |2i| = \sqrt{2^2} = 2
\]
Question

If \((x + iy)^3 = u + iv\), then show that

\[ \frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right) \]

Answer

\[ (x + iy)^3 = u + iv \]
\[ \Rightarrow x^3 + (iy)^3 + 3 \cdot x \cdot iy(x + iy) = u + iv \]
\[ \Rightarrow x^3 + i^3y^3 + 3x^2yi + 3xy^2i^2 = u + iv \]
\[ \Rightarrow x^3 - iy^3 + 3x^2yi - 3xy^2 = u + iv \]
\[ \Rightarrow \left(x^3 - 3xy^2\right) + i\left(3x^2y - y^3\right) = u + iv \]

On equating real and imaginary parts, we obtain

\[ u = x^3 - 3xy^2, \quad v = 3x^2y - y^3 \]
\[ \therefore \frac{u}{x} + \frac{v}{y} = \frac{x^3 - 3xy^2}{x} + \frac{3x^2y - y^3}{y} \]
\[ = \frac{x\left(x^2 - 3y^2\right)}{x} + \frac{y\left(3x^2 - y^3\right)}{y} \]
\[ = x^2 - 3y^2 + 3x^2 - y^2 \]
\[ = 4x^2 - 4y^2 \]
\[ = 4\left(x^2 - y^2\right) \]

\[ \therefore \frac{u}{x} + \frac{v}{y} = 4\left(x^2 - y^2\right) \]

Hence, proved.
Question

If \( \alpha \) and \( \beta \) are different complex numbers with \( |\beta| = 1 \), then find \( \left| \frac{\beta - \alpha}{1 - \overline{\alpha} \beta} \right| \).

Answer

Let \( \alpha = a + ib \) and \( \beta = x + iy \)

It is given that, \( |\beta| = 1 \)

\[
\therefore \sqrt{x^2 + y^2} = 1
\]

\[
\Rightarrow x^2 + y^2 = 1 \quad \text{... (i)}
\]

\[
\left| \frac{\beta - \alpha}{1 - \overline{\alpha} \beta} \right| = \left| \frac{(x + iy) - (a + ib)}{1 - (a - ib)(x + iy)} \right|
\]

\[
= \left| \frac{(x - a) + i(y - b)}{1 - (ax + ay - i bx + by)} \right|
\]

\[
= \left| \frac{(x - a) + i(y - b)}{(1 - ax - by) + i(bx - ay)} \right|
\]

\[
= \left| \frac{(x - a) + i(y - b)}{\sqrt{(1 - ax - by)^2 + (bx - ay)^2}} \right|
\]

\[
= \frac{\sqrt{(x - a)^2 + (y - b)^2}}{\sqrt{(1 - ax - by)^2 + (bx - ay)^2}}
\]

\[
= \frac{\sqrt{x^2 + a^2 - 2ax + y^2 + b^2 - 2by}}{\sqrt{1 + a^2 x^2 + b^2 y^2 - 2ax + 2abxy - 2by + b^2 x^2 + a^2 y^2 - 2abxy}}
\]
The above method is very long and stupid. There is a very short way of doing this.

Understand the fundamentals.... \(| Z | = 1 \Rightarrow Z (\bar{Z}) = 1 \)

There is a typing issue here the \(Z\) bar symbol is not there. A bar symbol is available \(\bar{A}\)

So writing \(Z\) bar as \(\bar{Z}\)

In the problem it is given \(| \beta | = 1\) so \(\beta (\bar{\beta}) = 1 \)

The denominator \(1 - \beta (\bar{\alpha})\) can be written as \(\beta (\bar{\beta}) - \beta (\bar{\alpha})\)

Take \(\beta\) common so \(\beta(\bar{\beta} - \bar{\alpha})\)

Now \(| \beta - \alpha | = | \bar{\beta} - \bar{\alpha} |\) You should know that \(| Z \pm \bar{A} | = | \bar{Z} \pm A |\)

Or say \(| A - Z | = | \bar{A} - \bar{Z} |\) and so on

So our problem simplifies as \(1 / | \beta |\) which is 1

:-{D
Question

Find the number of non-zero integral solutions of the equation $|1-i|^x = 2^x$.

Answer

$|1-i|^x = 2^x$
$\Rightarrow \left(\sqrt{1^2 + (-1)^2}\right)^x = 2^x$
$\Rightarrow \left(\sqrt{2}\right)^x = 2^x$
$\Rightarrow 2^{\frac{x}{2}} = 2^x$
$\Rightarrow \frac{x}{2} = x$
$\Rightarrow x = 2x$
$\Rightarrow 2x - x = 0$
$\Rightarrow x = 0$

Thus, 0 is the only integral solution of the given equation. Therefore, the number of non-zero integral solutions of the given equation is 0.

Question

If $(a + ib)(c + id)(e + if)(g + ih) = A + iB$, then show that $(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$.

Answer

$(a + ib)(c + id)(e + if)(g + ih) = A + iB$
$\Rightarrow |(a + ib)(c + id)(e + if)(g + ih)| = |A + iB|$
$\Rightarrow |(a + ib)||c + id||e + if||g + ih| = |A + iB|$

$\Rightarrow |z_1z_2| = |z_1||z_2|$

On squaring both sides, we obtain
$(a^2 + b^2)(c^2 + d^2)(e^2 + f^2)(g^2 + h^2) = A^2 + B^2$

Hence, proved.
Question

\[
\left( \frac{1+i}{1-i} \right)^m = 1
\]

If \( \left( \frac{1+i}{1-i} \right)^m = 1 \), then find the least positive integral value of \( m \).

Answer

\[
\left( \frac{1+i}{1-i} \right)^m = 1
\]

\[
\Rightarrow \left( \frac{1+i}{1-i} \times \frac{1+i}{1+i} \right)^m = 1
\]

\[
\Rightarrow \left( \frac{(1+i)^2}{1^2+1^2} \right)^m = 1
\]

\[
\Rightarrow \left( \frac{1^2 + i^2 + 2i}{2} \right)^m = 1
\]

\[
\Rightarrow \left( \frac{1 - 1 + 2i}{2} \right)^m = 1
\]

\[
\Rightarrow \left( \frac{2i}{2} \right)^m = 1
\]

\[
\Rightarrow i^m = 1
\]

\[
\therefore m = 4k, \text{ where } k \text{ is some integer.}
\]

Therefore, the least positive integer is 1.

Thus the least positive integral value of \( m \) is 4.
Question

\[i^{49} + i^{58} + i^{69} + i^{110} = i^{4\times12} + i^{4\times17} + i^{4\times22} + i^{4\times27} \times i \times i^2\]
\[= i + 1 + 1 + i + 1 \times i^2\]
\[= i + 1 + i - 1\]
\[= 2i\]

\[\therefore i^{49} + i^{58} + i^{69} + i^{110} = 2i\]

\[i^{30} + i^{80} + i^{120} = i^{4\times7} + i^{4\times20} + i^{4\times30}\]
\[= 1 \times i^2 + 1 + 1\]
\[= -1 + 1 + 1\]
\[= 1\]

\[\therefore i^{30} + i^{80} + i^{120} = 1\]

\[i + i^2 + i^3 + i^4 = 1 + (-1) + (-i) + 1\]
\[= 0\]

\[\therefore i + i^2 + i^3 + i^4 = 0\]
\[ i^{5} + i^{10} + i^{15} = i^{4} \times 1 + i^{4} \times 2 \times i^{2} + i^{4} \times 3 \times i^{3} = 1 \times 1 + 1 \times i^{2} + 1 \times i^{3} = i - 1 - i = -1 \]

\[ i^{592} + i^{590} + i^{588} + i^{584} = \frac{i^{4} \times 148 + i^{4} \times 147 + i^{4} \times 146 + i^{4} \times 145 \times i^{2}}{i^{4} \times 145 + i^{4} \times 144 + i^{4} \times 143 \times i^{2}} = \frac{1 + 1 \times i^{2} + 1 + 1 \times i^{2} + 1}{1 \times i^{2} + 1 + 1 \times i^{2} + 1 + 1 \times i^{2}} = \frac{1 - 1 + 1 - 1}{-1 + 1 - 1} = 1 \]

\[ i^{592} + i^{590} + i^{588} + i^{584} = -1 \]

\[ 1 + i^{2} + i^{4} + i^{6} + i^{8} + \ldots + i^{20} = 1 + i^{2} + i^{4} \times 1 \times i^{2} + i^{4} \times 2 \times i^{2} + i^{4} \times 3 \times i^{2} + i^{4} \times 4 \times i^{2} = 1 - 1 + 1 \times i^{2} + 1 + 1 \times i^{2} + 1 + 1 \times i^{2} + 1 + 1 \times i^{2} = 1 - 1 + 1 - 1 + 1 - 1 + 1 - 1 = 1 \]
To recall standard integrals

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$\int f(x) , dx$</th>
<th>$f(x)$</th>
<th>$\int f(x) , dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^n$</td>
<td>$\frac{x^{n+1}}{n+1}$ $(n \neq -1)$</td>
<td>$\frac{g(x)}{n+1}$</td>
<td>$\frac{x^{n+1}}{n+1}$ $(n \neq -1)$</td>
</tr>
<tr>
<td>$\frac{1}{x}$</td>
<td>$\ln</td>
<td>x</td>
<td>$</td>
</tr>
<tr>
<td>$e^x$</td>
<td>$e^x$</td>
<td>$a^x$</td>
<td>$\frac{a^x}{\ln a}$ $(a &gt; 0)$</td>
</tr>
<tr>
<td>$\sin x$</td>
<td>$-\cos x$</td>
<td>$\sinh x$</td>
<td>$\cosh x$</td>
</tr>
<tr>
<td>$\cos x$</td>
<td>$\sin x$</td>
<td>$\cosh x$</td>
<td>$\sinh x$</td>
</tr>
<tr>
<td>$\tan x$</td>
<td>$-\ln</td>
<td>\cos x</td>
<td>$</td>
</tr>
<tr>
<td>$\sec x$</td>
<td>$\ln</td>
<td>\sec x + \tan x</td>
<td>$</td>
</tr>
<tr>
<td>$\csc x$</td>
<td>$\ln</td>
<td>\csc x</td>
<td>$</td>
</tr>
<tr>
<td>$\sin^2 x$</td>
<td>$\frac{x}{2} - \frac{2x}{4}$</td>
<td>$\sinh^2 x$</td>
<td>$\frac{\sinh^2 x}{4} - \frac{x}{2}$</td>
</tr>
<tr>
<td>$\cos^2 x$</td>
<td>$\frac{x}{2} + \frac{2x}{4}$</td>
<td>$\cosh^2 x$</td>
<td>$\frac{\cosh^2 x}{4} + \frac{x}{2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>$\int f(x) , dx$</th>
<th>$f(x)$</th>
<th>$\int f(x) , dx$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{a^2 + x^2}$</td>
<td>$\frac{1}{a} \tan^{-1} \frac{x}{a}$</td>
<td>$0 &lt;</td>
<td>x</td>
</tr>
<tr>
<td>$(a &gt; 0)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{\sqrt{a^2 - x^2}}$</td>
<td>$\sin^{-1} \frac{x}{a}$</td>
<td>$-a &lt; x &lt; a$</td>
<td></td>
</tr>
<tr>
<td>$(a &gt; 0)$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sqrt{a^2 - x^2}$</td>
<td>$\frac{x^2}{2} \left[ \sin^{-1} \left( \frac{x}{a} \right) + \sqrt{\frac{a^2-x^2}{a^2}} \right]$</td>
<td>$\frac{x^2}{2} \left[ \sinh^{-1} \left( \frac{x}{a} \right) + \sqrt{\frac{a^2-x^2}{a^2}} \right]$</td>
<td></td>
</tr>
<tr>
<td>$(x &gt; a &gt; 0)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Some series expansions -

\[
\pi = \frac{4}{1} - \frac{4}{3} + \frac{4}{5} - \frac{4}{7} + \frac{4}{9} - \frac{4}{11} + \frac{4}{13} - \ldots
\]

\[
\pi = \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{9} - \ldots
\]

\[
\pi = \sqrt{12} \left( 1 - \frac{1}{3^2} + \frac{1}{5^2} - \frac{1}{7^2} + \frac{1}{9^2} + \ldots \right)
\]

\[
\frac{\pi^2}{6} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \sum_{n=1}^{\infty} \frac{1}{n^2}
\]

\[
\int_0^{\pi} \log \sin x \, dx = -\frac{\pi}{2} \log 2 = \frac{\pi}{2} \log \frac{1}{2}
\]

Solve a series problem

If \[\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \text{ upto } \infty = \frac{\pi^2}{6}\], then value of

\[
\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \text{ up to } \infty
\]

(a) \[\frac{\pi^2}{4}\]  (b) \[\frac{\pi^2}{6}\]  (c) \[\frac{\pi^2}{8}\]  (d) \[\frac{\pi^2}{12}\]

**Ans. (c)**

**Solution** We have

\[
\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots \text{ upto } \infty
\]

\[
= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} + \frac{1}{6^2} + \ldots \text{ upto } \infty
\]

\[
= -\frac{1}{2^2} \left[ \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots \right]
\]

\[
= \frac{\pi^2}{6} - \frac{1}{4} \left( \frac{\pi^2}{6} \right) = \frac{\pi^2}{8}
\]

\[
1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \ldots \text{ upto } \infty = \frac{\pi^2}{12}
\]

\[
\frac{1}{2^2} + \frac{1}{4^2} + \frac{1}{6^2} + \ldots \text{ upto } \infty = \frac{\pi^2}{24}
\]

CBSE Math Survival Guide - Complex Numbers by Prof. Subhashish Chattopadhyay SKMClasses
Bangalore Useful for IIT-JEE, I.Sc. PU-II, Boards, IGCSE IB AP-Mathematics and other exams
\[
\sin \sqrt{x} = 1 - \frac{x}{3!} + \frac{x^3}{5!} - \frac{x^5}{7!} + \frac{x^7}{9!} - \frac{x^9}{11!} + \ldots
\]

\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \ldots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}
\]

\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \ldots = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}
\]

\[
\cosh x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots = \sum_{k=0}^{\infty} \frac{x^{2k}}{(2k)!}
\]

\[
\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}
\]

\[
\tan^{-1} x = x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \ldots \quad (-1 \leq x < 1)
\]

\[
\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \ldots + \frac{2^{2n}(2^{2n}-1)B_{2n}x^{2n-1}}{(2n)!} + \ldots \quad |x| < \frac{\pi}{2}
\]

\[
\sec x = 1 + \frac{x^2}{2} + \frac{5x^4}{24} + \frac{61x^6}{720} + \ldots + \frac{B_{2n}x^{2n}}{(2n)!} + \ldots \quad |x| < \frac{\pi}{2}
\]

\[
\csc x = 1 + \frac{x}{6} + \frac{7x^3}{360} + \frac{31x^5}{15120} + \ldots + \frac{2(2^{2n}-1)B_{2n}x^{2n-1}}{2n!} + \ldots \quad 0 < |x| < \pi
\]

\[
\cot x = 1 - \frac{x}{3} - \frac{x^3}{45} - 2\frac{x^5}{945} - \ldots - \frac{2^{2n}B_{2n}x^{2n-1}}{(2n)!} + \ldots \quad 0 < |x| < \pi
\]
\[
\begin{align*}
\tan x &= x + \frac{x^3}{3} + \frac{2x^5}{15} + \cdots \\
\sec x &= 1 + \frac{x^2}{2} + \frac{5x^4}{4} + \cdots \\
\log (\cos x) &= -\frac{x^2}{2} - \frac{2x^4}{4} - \cdots \\
\log (1 + \sin x) &= x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{12} + \cdots \\

\sin^{-1} x &= x + \frac{1}{2} \frac{x^3}{3} + \frac{1}{2} \frac{1}{5} \frac{x^5}{2} + \frac{1}{2} \frac{1}{5} \frac{1}{7} \frac{x^7}{2} + \cdots \quad |x| < 1 \\
\cos^{-1} x &= \frac{\pi}{2} - \sin^{-1} x \\
&= \frac{\pi}{2} - \left( x + \frac{1}{2} \frac{x^3}{3} + \frac{1}{2} \frac{1}{5} \frac{x^5}{2} + \frac{1}{2} \frac{1}{5} \frac{1}{7} \frac{x^7}{2} + \cdots \right) \quad |x| < 1 \\
\tan^{-1} x &= \begin{cases} \\
& x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \quad |x| < 1 \\
& \pm \frac{\pi}{2} - \frac{1}{3x^3} + \frac{1}{5x^5} - \frac{1}{7x^7} + \cdots \quad \begin{cases} + \text{if } x \geq 1 \\
- \text{if } x \leq -1 \end{cases} \\
\end{cases} \\
\sec^{-1} x &= \cos^{-1} \left( \frac{1}{x} \right) \\
&= \frac{\pi}{2} - \left[ x + \frac{1}{2} \frac{x^3}{3} + \frac{1}{2} \frac{1}{5} \frac{x^5}{2} + \frac{1}{2} \frac{1}{5} \frac{1}{7} \frac{x^7}{2} + \cdots \right] \quad |x| > 1 \\
csc^{-1} x &= \sin^{-1} \left( \frac{1}{x} \right) \\
&= \frac{1}{x} + \frac{1}{2} \frac{x^3}{3} + \frac{1}{2} \frac{1}{5} \frac{x^5}{2} + \frac{1}{2} \frac{1}{5} \frac{1}{7} \frac{x^7}{2} + \cdots \quad |x| > 1 \\
cot^{-1} x &= \frac{\pi}{2} - \tan^{-1} x \\
&= \begin{cases} \\
& x - \frac{x^3}{3} + \frac{x^5}{5} - \frac{x^7}{7} + \cdots \quad |x| < 1 \\
& px + \frac{1}{3x^3} + \frac{1}{5x^5} + \cdots \quad \begin{cases} p = 0 \text{ if } x \geq 1 \\
 p = 1 \text{ if } x \leq -1 \end{cases} \\
\end{cases}
\end{align*}
\]
\[ e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{n=0}^{\infty} \frac{x^n}{n!} \]

\[ \ln x = 2 \left[ \frac{x-1}{x+1} + \frac{1}{3} \left( \frac{x-1}{x+1} \right)^3 + \frac{1}{5} \left( \frac{x-1}{x+1} \right)^5 + \ldots \right] \]
\[ = 2 \sum_{n=1}^{\infty} \frac{1}{2n-1} \left( \frac{x-1}{x+1} \right)^{2n-1} \quad (x > 0) \]

\[ \ln x = x - \frac{1}{2} \left( \frac{x-1}{x} \right)^2 + \frac{1}{3} \left( \frac{x-1}{x} \right)^3 + \ldots \]
\[ = \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left( \frac{x-1}{x} \right)^n \quad (0 < x \leq 2) \]

\[ \ln (1 + x) = x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots \]
\[ = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{1}{n} x^n \quad (|x| < 1) \]

\[ \log_e (1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \ldots \infty (-1 \leq x < 1) \]

\[ \log_e (1 + x) - \log_e (1 - x) = \]

\[ \log_e \left( \frac{1 + x}{1 - x} \right) = 2 \left( x + \frac{x^3}{3} + \frac{x^5}{5} + \ldots \infty \right) (-1 < x < 1) \]

\[ \log_e \left( 1 + \frac{1}{n} \right) = \log_e \frac{n+1}{n} = 2 \left[ \frac{1}{2n+1} + \frac{1}{3(2n+1)^3} + \frac{1}{5(2n+1)^5} + \ldots \infty \right] \]

\[ \log_e (1 + x) + \log_e (1 - x) = \log_e (1 - x^2) = -2 \left( x^2 + \frac{x^4}{2} + \ldots \infty \right) (-1 < x < 1) \]

\[ \log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \ldots = \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots \]
Important Results

(i) \( \int_0^{\pi/2} \frac{\sin^n x}{\sin^n x + \cos^n x} \, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cos^n x}{\sin^n x + \cos^n x} \, dx \)

(b) \( \int_0^{\pi/2} \frac{\tan^n x}{1 + \tan^n x} \, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\tan^{n/2} x}{1 + \tan^n x} \, dx \)

(c) \( \int_0^{\pi/2} \frac{dx}{1 + \cot^n x} = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{1 + \cot^n x} \, dx \)

(d) \( \int_0^{\pi/2} \frac{\tan^n x}{\tan^n x + \cot^n x} \, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cot^n x}{\tan^n x + \cot^n x} \, dx \)

(e) \( \int_0^{\pi/2} \frac{\sec^n x}{\sec^n x + \cosec^n x} \, dx = \frac{\pi}{4} = \int_0^{\pi/2} \frac{\cosec^n x}{\sec^n x + \cosec^n x} \, dx \) where, \( n \in \mathbb{R} \)

(ii) \( \int_0^{\pi/2} \frac{a^{\sin^n x}}{a^{\sin^n x} + a^{\cos^n x}} \, dx = \int_0^{\pi/2} \frac{a^{\cos^n x}}{a^{\sin^n x} + a^{\cos^n x}} \, dx = \frac{\pi}{4} \)

(iii) (a) \( \int_0^{\pi/2} \log \sin x \, dx = \int_0^{\pi/2} \log \cos x \, dx = -\frac{\pi}{2} \log 2 \)

(b) \( \int_0^{\pi/2} \log \tan x \, dx = \int_0^{\pi/2} \log \cot x \, dx = 0 \)

(c) \( \int_0^{\pi/2} \log \sec x \, dx = \int_0^{\pi/2} \log \cosec x \, dx = \frac{\pi}{2} \log 2 \)

(iv) (a) \( \int_0^\infty e^{-ax} \sin bx \, dx = \frac{b}{a^2 + b^2} \)

(b) \( \int_0^\infty e^{-ax} \cos bx \, dx = \frac{a}{a^2 + b^2} \)

(c) \( \int_0^\infty e^{-ax} x^n \, dx = \frac{n!}{a^n + 1} \)
\[
\int \frac{dx}{\sqrt{x^2 - a^2}} = \ln \left( x + \sqrt{x^2 - a^2} \right) + C \\
\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln \left( x + \sqrt{x^2 + a^2} \right) + C \\
\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} \ln \left( \frac{x-a}{x+a} \right) + C \\
\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln \left( \frac{a+x}{a-x} \right) + C \\
\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \left( \frac{x}{a} \right) + C \\
\int \sqrt{a^2 + x^2} \, dx = \frac{x}{2} \sqrt{a^2 + x^2} + \frac{a^2}{2} \sinh^{-1} \left( \frac{x}{a} \right) + C \\
\int \sqrt{x^2 - a^2} \, dx = \frac{x}{2} \sqrt{x^2 - a^2} - \frac{a^2}{2} \cosh^{-1} \left( \frac{x}{a} \right) + C
\]
Good Luck to you for your Preparations, References, and Exams

All Other Books written by me can be downloaded from


Professor Subhashish Chattopadhyay

Learn more at http://skmclasses.weebly.com/iit-jee-home-tuitions-bangalore.html

Twitter - https://twitter.com/ZookeeperPhy

Facebook - https://www.facebook.com/IIT.JEE.by.Prof.Subhashish/

Blog - http://skmclasses.kinja.com